Reigate Park Primary, Ash Croft and Cottons Farm Primary Federation

Calculations Policy   September 2017

This policy has been written to ensure consistency and progression in calculation and reflects a whole school agreement. We believe that mental calculation should be seen as complementary to written recording and, as such, it is practised and secured alongside children's understanding and use of each method. Whichever method a child is working on, it must still be underpinned by a secure and appropriate knowledge of relevant number facts and place value, otherwise they will be unable to progress to the next method. Children are always encouraged to consider whether they can solve a calculation mentally, before relying upon a written method.

OVERALL AIMS

When children leave our schools they:

- Have a secure understanding of the four operations and place value.
- Can recall and apply number facts rapidly.
- Are able to use this knowledge and understanding to carry out calculations mentally.
- Make use of diagrams and informal notes to aid mental methods.
- Have an efficient, reliable and compact written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally.
- Can use these methods to solve a variety of routine and non-routine problems, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

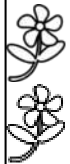
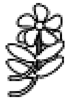
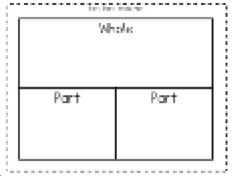
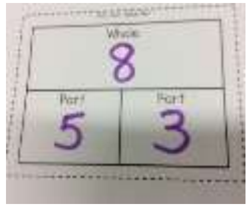
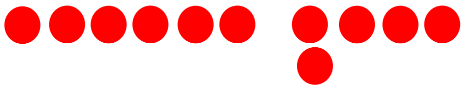
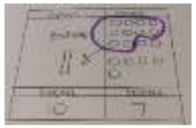
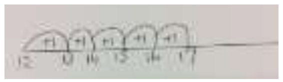
Multiplication ⟷ is inverse of ⟷ **Division**

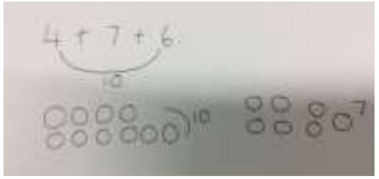
Multiplication ↓ is repeated ↓ **Addition**

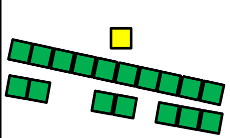
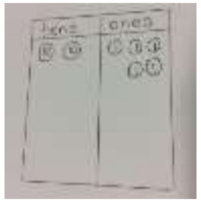
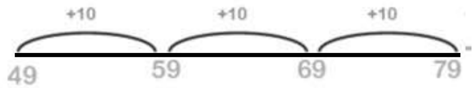
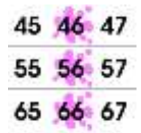
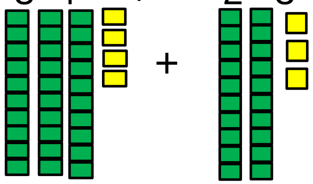
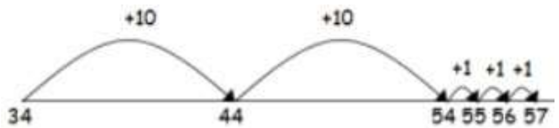
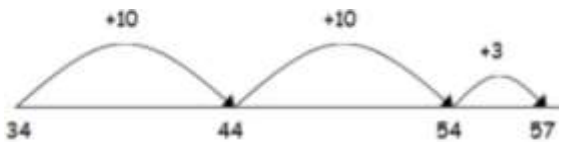
Division ↓ is repeated ↓ **Subtraction**

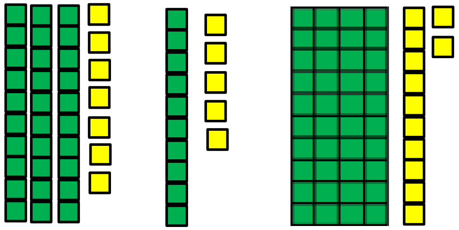
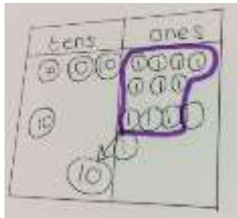
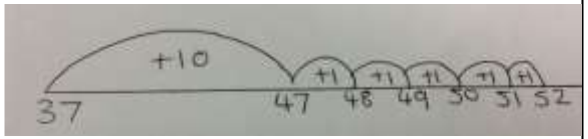
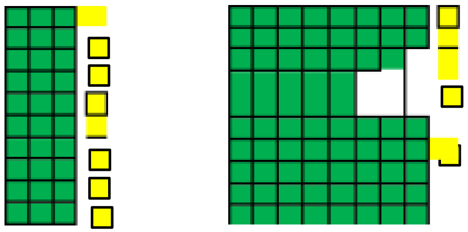
Addition ⟷ is inverse of ⟷ **Subtraction**

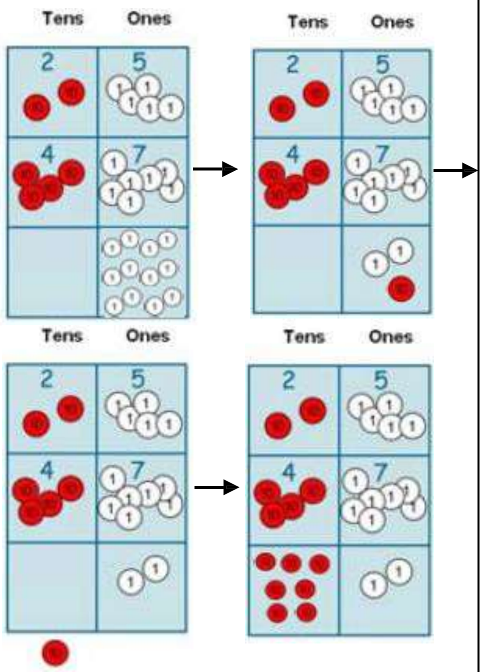
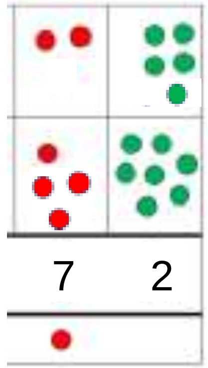
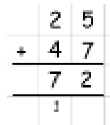
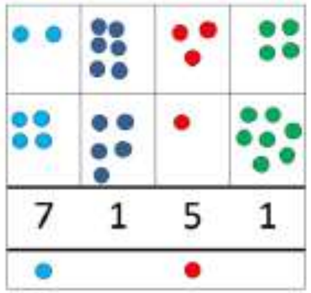
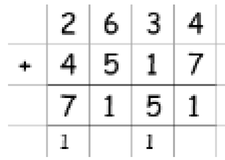
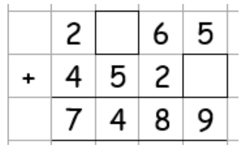
Addition

	Strategy & Key Skills	Understanding	Application		
			Concrete	**Pictorial**	**Abstract**
EYFS	*Combining two parts to make a whole: part- whole model.* •Count from 0 – 20. •Place numbers in order. •Say what is one more than a given number (to 20).	•Understand that addition is combining two sets of objects. •Beginning to use the symbols + and =. Key Vocab: More than, most, add, altogether.	  Use cubes/counters and bead strings to add two numbers together as a group. Start with the larger number on the bead string and then count on the smaller number 1 by 1 to find the answer.	  Draw pictures to add two numbers together. Use the whole part-part diagram to visualise what calculation should be completed. 	Use the part-part whole diagram to move into the abstract.  8 = 5 + 3 8 = 3 + 5 5 + 3 = 8
YEAR 1 (EYFS)	*Starting at the bigger number and counting on.* •Count, read and write numbers to 100 in numerals (to 20 in words). •Say what is one more than a given number (to 100). •Recall number bonds within 10.	•Relate addition to counting on. •Understand the effect of adding zero. •Understand and use the symbols + and =. •Understand what each digit represents in a teens number. Key Vocab: More than, most, add, count on, altogether, put together, plus, total.	 Start with the larger number on the bead string and then count on the smaller number 1 by 1 to find the answer. Add together two groups of objects. **Count out** the correct number of ones counters/base ten for each number and then add together to find the total. 6 + 5 = 	Add together two groups of objects. **Draw** the correct number of ones circles for each number and then add together to find the total. 6 + 5 =  Progress to **drawing** base ten/ones counters. Using the tens and ones grid. More able Chn to exchange ten ones for a ten.  First using a numbered line progressing to using a blank number line. Start at the larger number on the number line and count on in **ones.** 12 + 5 = 17 	12 + 5 = 17 Place the larger number in your head and count on the smaller number to find your answer. Use related addition facts, e.g. you know 5 + 2 = 7 so 15 + 2 = 17.

YEAR 2	*Adding three single digits.* • Read and write numbers to at least 100 in numerals and words. • Recall number bonds within and to 20 fluently. • Double any single digit number.	• Understand that addition is commutative (can be done in any order). • Understand that you can add 9 by adding 10 and taking 1 away. <u>Key Vocab:</u> More than, add, count on, altogether, put together, plus, total, sum.	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Make 10 with 2 of the digits (if possible) then add on the third digit. If making 10 is not possible then look for other facts, e.g. doubles, adding 9 etc. If no facts can be spotted, then children should start with the bigger number.	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 4 + 7 + 6 	Combine the two numbers that make 10 and then add on the remainder. $\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Children will progress to adding mentally without any jottings.

YEAR 2	*Adding two-digit numbers and ones/tens.* • Read and write numbers to at least 100 in numerals and words. • Recall number bonds within and to 20 fluently. • Derive facts to 100, e.g. 3 + 7 = 10 so 30 + 70 = 100.	• Understand place value in two-digit numbers and how zero is a place holder. • Understand that subtraction is the inverse of addition. <u>Key Vocab:</u> Add, altogether, plus, total, sum, inverse, partition, multiple of 10.	Using base 10/place value counters to add a two-digit number and ones. 21 + 2 =23   Move the tens and ones into the chart. Add the ones then add the tens.	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column Using number lines. 49 + 30 = 79  Using 100 square. 46 + 20 = 66 	49 + 30 = 79 Starting at 49, can you count in tens? How many tens do you need to add? Children could use their fingers to make sure they add the correct amount of tens. Children to use part-part whole diagram to help understand that subtraction is the inverse. See model above. 49 + 30 = 79 30 + 49 = 79 79 – 49 = 30 79 – 30 = 49
YEAR 2	*Adding two two-digit numbers (without crossing the tens boundary).* • Recall number bonds within and to 20 fluently. • Know doubles to 20 and use this to add near doubles, e.g. 14 + 14 = 28 so 15 + 14 = 29.	• Understand place value in two-digit numbers and how zero is a place holder. • Understand that subtraction is the inverse of addition. • Understand how to add 9, 19… And 11, 21… <u>Key Vocab:</u> Add, altogether, plus, total, sum, make, inverse, partition, multiple of 10, tens, ones, tens	1) Using base 10, children need to start at the bigger number and then combine the ones and then the tens. 3 4 + 2 3 = 57  2) Using place value counters/base ten and the tens and ones grids, children need to combine the ones, combine the tens and then add the two totals. 26 + 2 3 = 49 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column Children will use empty number lines, starting with the bigger number and counting on. They will do this adding individual tens and ones. 34 + 23 = 57  	Combine the units and the tens. 34 + 23 = 57 50 + 7 OR 34 + 23 34 + 20 = 54 + 3 = 57 Children to add the numbers the other way round to check. 23 + 30 = 53 + 4 = 57 Children will progress to adding mentally without any jottings with whichever method they prefer.

YEAR 2	*Adding two two-digit numbers (including crossing the tens boundary).* • Recall number bonds within and to 20 fluently. • Know doubles to 20 and use this to add near doubles, e.g. 14 + 14 = 28 so 15 + 14 = 29.	• Understand place value in two-digit numbers and how zero is a place holder. • Understand that subtraction is the inverse of addition. • Understand how to add 9, 19… And 11, 21… <u>Key Vocab:</u> Add, altogether, plus, total, sum, make, inverse, partition, multiple of 10, tens, ones, tens	Start with base 10 and combine ones to make a 10 tower 3 7 + 1 5 = 5 2  Then children should move onto substituting with place value counters, exchange the 10 ones, for a tens counter. 3 7 + 1 5 = 5 2 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column Using an empty number line. First by ending in 10's and 1's then progressing to using the model above assessing multiples of 10 etc. 37 + 15 = 52 	Combine the ones and the tens. 37 + 15 = 52 40 + 12 OR 37 + 15 37 + 10 = 47 + 5 = 52 Children to add the numbers the other way round to check. 15 + 30 = 45 + 7 = 52 Children will progress to adding mentally without any jottings with whichever method they prefer.
YEAR 3	*Adding two and three-digit numbers (including crossing the hundreds boundary).* • Count, read and write numbers to 1000. • Find 10 or 100 more than a given number. • Know doubles to 50 and use these to add near doubles.	• Understand place value in three-digit numbers. • Understand how the inverse can be used to check answers. <u>Key Vocab:</u> Add, more, plus, make, altogether, total, double, most, count on, addend, sum, tens, ones, partition, addition, tens boundary, hundreds boundary, increase.	Using base 10 and place value counters. 3 8 + 8 6  	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column Using an empty number line, children count on from the largest number. 86 + 38 = 124 	Combine the units, tens and hundreds. 86 + 38 = 124 110 + 14 Use the inverse to check.

YEAR 3	*Column method – up to 3 digits.* • Count, read and write numbers to 1000. • Be able to add mentally: - two two-digit numbers - a three-digit number and ones - a three-digit number and tens - a three digit number and hundreds.	• Understand place value in three-digit numbers. • Understand how the inverse can be used to check answers. • Use rounding to estimate and check answers. <u>Key Vocab:</u> sum, digits, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry.	Using base 10 and then place value counters on laminated place value grids. **Progress to grids with hundred columns.** 25 + 47 Estimate: 30 + 50 = 80 	After practically using the base 10 blocks and place value counters, children can **draw** the counters to help them to the calculation. **Progress to grids with hundred columns.** 25 + 47 	Children can move onto the compact column method. Carry digits are recorded below the line. In this case one ten is carried. **Progress to grids with hundred columns.** 
YEAR 4	*Column method - up to 4 digits.* • Count, read and write numbers beyond 1000. • Find 1000 more than a given number	• Understand place value in four-digit numbers. • Understand how the inverse can be used to check answers. • Use rounding to estimate and check answers. <u>Key Vocab:</u> Addend, sum, carry, hundreds, thousands.	As above but moving straight to place value counters. Children to use them on laminated place value grids (Th H T O).	As above but with four-digit numbers. 2634 + 4517 = 7151 	Children should be using rounding to estimate and the inverse to check. 

YEAR 4 AND BEYOND

Column method- decimals.

- Count, read and write numbers beyond 1,000,000 (up to 10,000,000 for Year 6).
- Add mentally increasingly larger numbers.

- Understand place value in numbers beyond four-digits (up to 10,000,000 for Year 6).
- Understand how the inverse can be used to check answers.
- Use rounding to estimate and check answers.

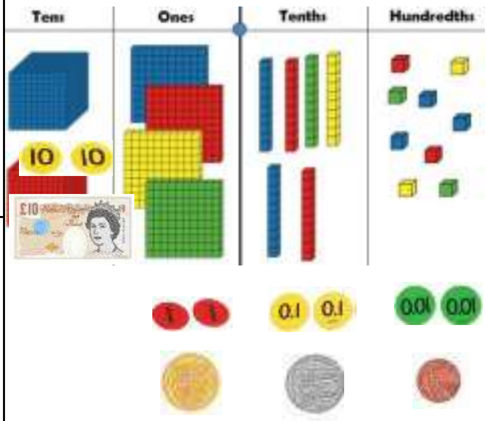
<u>Key Vocab:</u>
Carry, tenths, hundredths, decimal.

<u>Upper KS2</u>
Understand the use of brackets and the associative law of addition.
e.g. (a + b) + c = a + (b + c)
(1 + 2) + 5 = 1 + (2 + 5)

Begin to understand the order of operations – BODMAS.
B = Brackets
O = Orders (powers and square roots)
DM = Division and Multiplication
AS = Addition and Subtraction

e.g. 50 + 15 ÷ 5 (÷ before +)
50 - 3 = 53

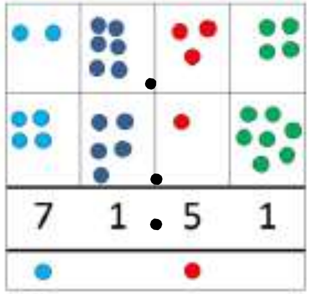
Concreate resources should be used in the following way; Start with base 10 (reassigning the values) on laminated place value grids (T O . t h) in the same way as the previous page and then move to place value counters and using money.



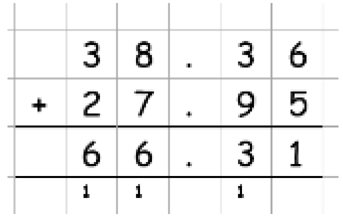
Children should practise adding numbers with both the same and different number of decimal places.

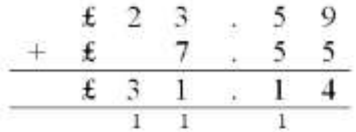
As per the previous page but with decimals.

26.34 + 45.17 = 71.51

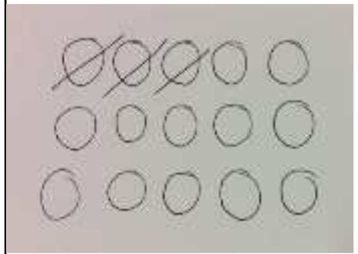
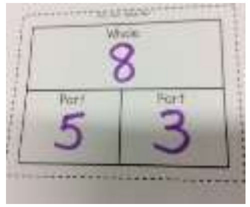
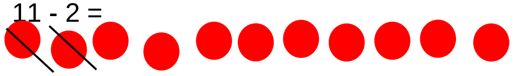
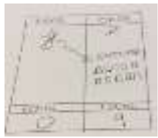
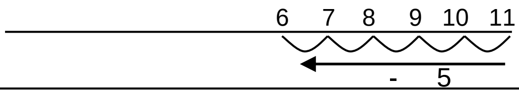


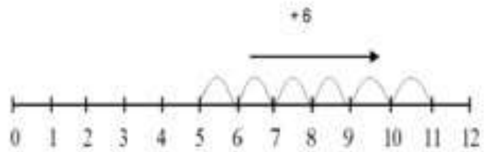
Children should be using rounding to estimate and the inverse to check.

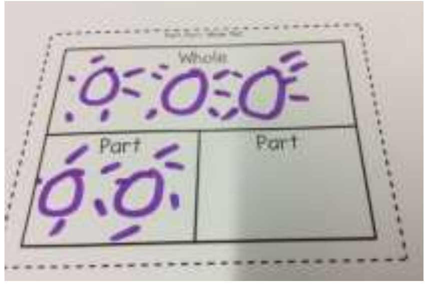
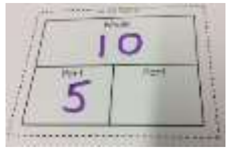
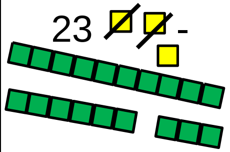
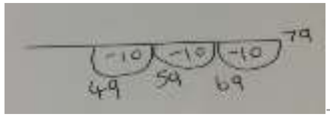
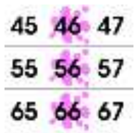


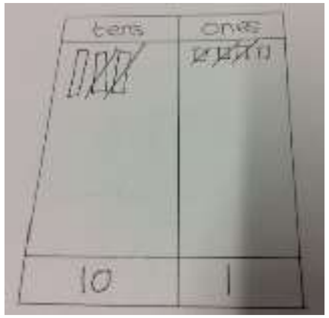
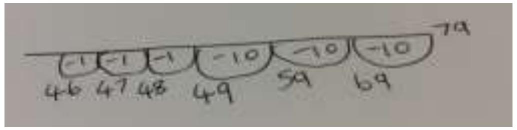
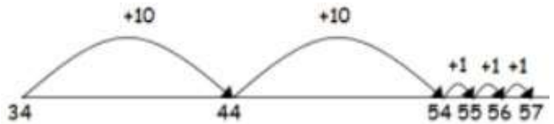


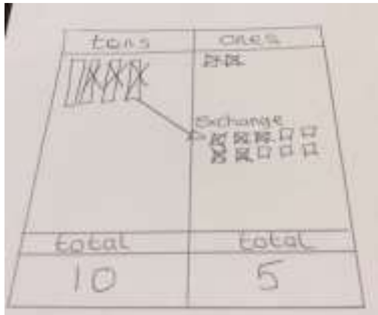
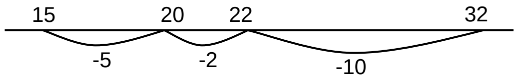
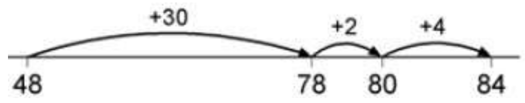
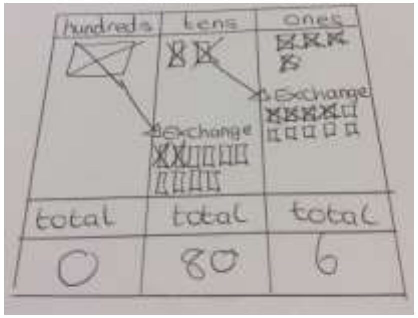
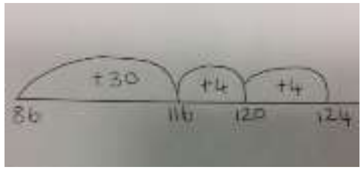
Subtraction

	Strategy & Key Skills	Understanding	Application		
			Concrete	**Pictorial**	**Abstract**
EYFS	*Taking away ones* • Count from 0 – 20. • Place numbers in order. • Say what is one less than a given number (to 20).	• Understands subtraction as removing or hiding objects (taking away) • Beginning to use the symbols - and =. Key Vocab: Less than, least, subtract, fewer.	6 - 4 = 2 Count out the correct number of counters/cubes and physically take the correct number away before counting how many are left.	Cross out drawn objects to show what has been taken away. 15 – 3 = 2 	5 – 3 = 2 6 = 8 – 2 Use the part-part whole diagram to move into the abstract. 
YEAR 1 (EYFS)	*Counting back* • Count, read and write numbers to 100 in numerals (to 20 in words). • Say what is one less than a given number (to 100). • Recall number bonds and related subtraction facts within 10.	• Relate subtraction to counting back. • Understand the effect of subtracting zero. • Understand and use the symbols - and =. • Understand what each digit represents in a teens number. Key Vocab: Take, take away, less, minus, subtract, leaves, between, fewer/less than, least, count back ,how many left.	Make the larger number in your Subtraction calculation. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters/cubes and move them away from the group as you take them away counting backwards as you go. 	Cross out drawn objects to show what has been taken away. **Draw** the correct number of ones circles and cross out the required amount to find how many are left. 11 - 2 =  Progress to **drawing** base ten/ones counters. Using the tens and ones grid. More able Chn to exchange ten for ten ones.  Count back on a number line or number track. Start at the bigger number and count back the smaller number showing the jumps on the number line. 11 – 5 = 6 	13 - 4 Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Use related subtraction facts, e.g. you know 5 - 2 = 3 so 15 - 2 = 13. 14 - $\square$ = 11 15 = 17 - $\square$

YEAR 1	*Find the difference* • Count, read and write numbers to 100 in numerals (to 20 in words). • Say what is one less than a given number (to 100). • Recall number bonds and related subtraction facts within 10.	• Relate subtraction to counting on to find the difference. • Understand the effect of subtracting zero. • Understand and use the symbols - and =. • Understand what each digit represents in a teens number. <u>Key Vocab:</u> Take, take away, less, minus, subtract, leaves, difference between, how many more, least, count on.	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference 	Count on to find the difference. Firstly using a numbered line, then using a blank number line. 11 – 5 = 6 	Hannah has 9 sandwiches, Helen has 7 sandwiches. Find the difference between the number of sandwiches. 9 – 7 Children to start at the 7 and count on. They could do this on their fingers to help them. It is worth talking to children about the two methods of subtraction at this point and which should be used when. When the numbers are close together, then it is easier to find the difference. However when you are only taking away small amounts, e.g. 14 – 3, then counting back is easier.

YEAR 1 and YEAR 2	*Part-Part Whole Model* • Recall number bonds to 10. • Begin to recall number bonds and related subtraction facts within and to 20.	• Understand that subtraction is <u>NOT</u> commutative (can be done in any order). • Understand that subtraction is the inverse of addition. <u>Key Vocab:</u> Take, take away, less, minus, subtract, leaves, difference between, how many more, how many left, least, count on.	Link to addition- use the part-part whole model to help explain the inverse between addition and subtraction If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 = Look here at why subtraction is not commutative. The 10 needs to go first in the subtraction because it is the whole – there are not enough cubes if you rearrange the calculation.	Use a pictorial representation of objects to show the part-part whole model. 	Move to using numbers within the part whole model.  13 - ☐ = 6 Year 2 - Children should also now be able to write the 4 calculations from a part-part whole diagram.
YEAR 2	*Subtracting ones/tens from two-digit numbers.* • Read and write numbers to at least 100 in numerals and words. • Recall number bonds within and to 20 fluently. • Derive facts to 100, e.g. 10 – 7 = 3 so 100 – 70 = 30.	• Understand place value in two-digit numbers and how zero is a place holder. Key Vocab: Decrease, take, take away, less, minus, <u>subtract, leaves</u>, how many more, how many fewer/less than, most, least, count back , difference, count on, partition, tens, ones.	Using base 10/ place value counters subtract ones/tens from a two-digit number. 23 - 2 = 21  Discuss how the tens do not change as long as there are enough ones in the starting number. 41 - 20 = 21 Discuss the fact that the ones will not change. 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column Starting at the **end** of the number line and counting **backwards.** 79 – 30 = 49  Using 100 square. 66 - 20 = 46 	49 - 30 = 19 Starting at 49, can you count back in tens? Children could use their fingers to make sure they subtract the correct amount of tens. Children to use part-part whole diagram to help understand that subtraction is the inverse. 49 + 30 = 79 30 + 49 = 79 79 – 49 = 30 79 – 30 = 49

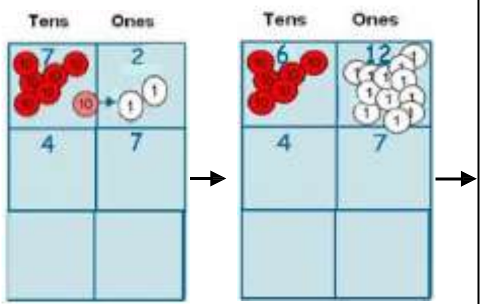
YEAR 2	*Subtracting from a two-digit number (without regrouping).* • Recall number bonds within and to 20 fluently. • Know doubles to 20 and use this to subtract near doubles, e.g. 14 + 14 = 28 so 29 – 14 = 15.	• Understand place value in two-digit numbers and how zero is a place holder. • Understand that addition is the inverse of subtraction. Key Vocab: Decrease, take, take away, less, minus, subtract, leaves, fewer/less than, least, count back , exchange difference, partition, tens, ones	1) Using base 10 or place value counters, children need to make the bigger number and place in the tens and ones chart. Then physically subtract each one, one at a time and then each ten one at a time. 3 4 - 2 3 = 11 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column. Children will use empty number lines. 1) Starting with the first number and counting back. They will do this in individual ones and tens jumps and progress to jumping in groups. 79 - 33 = 46  2) Counting on to find the difference. 57 - 34 = 23  At this point, discuss with the children which method they find easier. If numbers are closer together, then method 2 (finding the difference) is generally easier. However if they are only subtracting small amounts, then method 1 (counting back) is easier.	Counting back. 57 - 23 57 - 3 = 54 - 20 = 34 OR Comparing the tens and the ones of each number. 57 - 23 = 34 30 + 4 Use the inverse to check. 34 + 23 = 57 Children will progress to subtracting mentally without any jottings with whichever method they prefer.

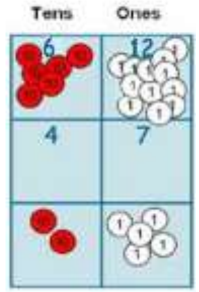
YEAR 2	*Subtracting two two-digit numbers (including regrouping).* • Recall number bonds within and to 20 fluently. • Know doubles to 20 and use this to subtract near doubles, e.g. 14 + 14 = 28 so 29 – 14 = 15.	• Understand place value in two-digit numbers and how zero is a place holder. • Understand that addition is the inverse of subtraction. <u>Key Vocab:</u> Decrease, take, take away, less, minus, subtract, leaves, fewer/less than, exchange, least, count back , difference, partition, tens, ones.	Using base 10 or place value counters. Exchange a 10 for 10 ones. 4 2 - 2 7 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column. 1) Using an empty number line to subtract two-digit numbers. Start by subtracting in 10's and1's then progressing to using model below. 32 - 17 = 15  2) Using a number line to find the difference through counting on. 84 – 48 = 36  Continue to discuss which method the children find easier as per above.	Use the inverse to check. 27 + 15 = 42 Children will progress to subtracting mentally without any jottings.
YEAR 3	*Subtracting from two and three-digit numbers (including regrouping).* • Count, read and write numbers to 1000. • Find 10 or 100 less than a given number. • Know doubles to 50 and use these to subtract near doubles.	• Understand place value in three-digit numbers. • Understand how the inverse can be used to check answers. <u>Key Vocab:</u> Decrease, subtract, how many more, how many fewer/less than, most, least, count back, difference, count on, partition, tens, ones, tens boundary, exchange.	Using base 10 or place value counters. Exchange a 100 for 10 tens and a 10 for 10 ones. 124 - 38 	**Draw** the base ten/ place value counters on own laminated grid. See example on concrete column. Using an empty number line, children count on. 124 – 86 = 38 	Use the inverse to check. 38 + 86 = 12

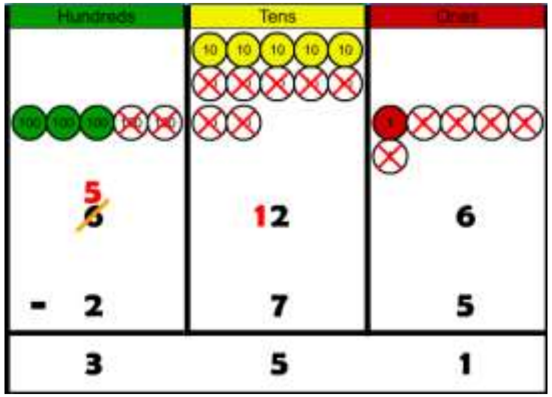
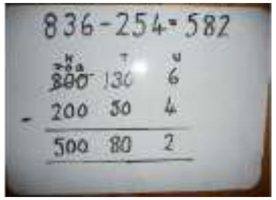
<table>
<tr>
<td>YEAR 3</td>
<td>Column method – up to 3 digits.

•Count, read and write numbers to 1000.
•Be able to subtract mentally:
- two two-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three digit number and hundreds.</td>
<td>•Understand place value in three-digit numbers.
•Understand how the inverse can be used to check answers.
•Use rounding to estimate and check answers.

Key Vocab:
Subtract, difference, digits, tens, ones, partition, addition, column, tens boundary, hundreds boundary, decrease, vertical, exchange.</td>
<td>Using base 10 and then place value counters on laminated place value grids. Progress to grids with hundred columns.

72 - 47 Estimate: 70 - 50 = 20




“I can’t take 7 ones from 2 ones so I need to exchange one of my tens for ten ones.”</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them solve subtractions. They will need to cross the counters out

</td>
<td>

Partitioning the number into clear place value columns.

Moving forward the children use a more compact method.


NOTE: By this stage children should be able to subtract two two-digit numbers mentally and as soon as this is the case they would not need a written method. They should be encouraged to use this method for bigger numbers only.</td>
</tr>
<tr>
<td>YEAR 4</td>
<td>Column method - up to 4 digits.

•Count, read and write numbers beyond 1000.
•Find 1000 less than a given number</td>
<td>•Understand place value in four-digit numbers.
•Understand how the inverse can be used to check answers.
•Use rounding to estimate and check answers.

Key Vocab:
Exchange, hundreds, thousands.</td>
<td>As above but moving straight to place value counters. Children to use them on laminated place value grids (Th H T O).</td>
<td>As above but with four-digit numbers.</td>
<td>Children should be using rounding to estimate and the inverse to check.

7   6 5
- 4 5 2
3 4 3 9</td>
</tr>
</table>

YEAR 4 AND BEYOND

Column method- decimals.

- Count, read and write numbers beyond 1,000,000 (up to 10,000,000 for Year 6).
- Subtract mentally increasingly larger numbers.

- Understand place value in numbers beyond four-digits (up to 10,000,000 for Year 6).
- Understand how the inverse can be used to check answers.
- Use rounding to estimate and check answers.

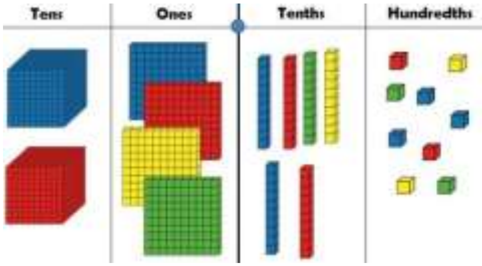
Key Vocab:
Exchange, tenths, hundredths, decimal.

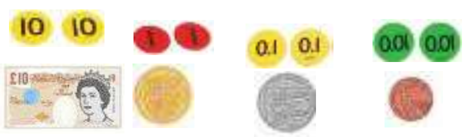
Upper KS2
Understand the use of brackets and the associative law of addition.
e.g. (a + b) + c = a + (b + c)
(1 + 2) + 5 = 1 + (2 + 5)

Begin to understand the order of operations – BODMAS.
B = Brackets
O = Orders (powers and square roots)
DM = Division and Multiplication
AS = Addition and Subtraction

e.g. 50 + 15 ÷ 5 (÷ before +)
50 - 3 = 53

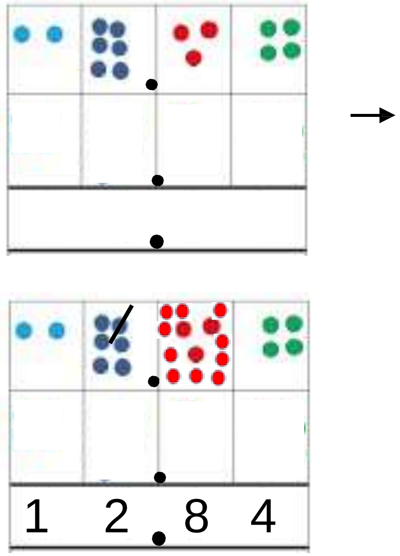
Concrete materials should be used when introducing the subtraction of decimals. Start with base 10 (reassigning the values) on laminated place value grids (T O . t h) in the same way as the previous page and then move to place value counters and using money.



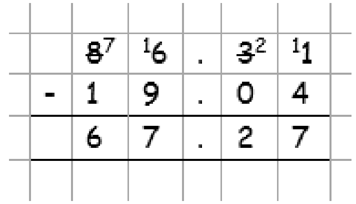


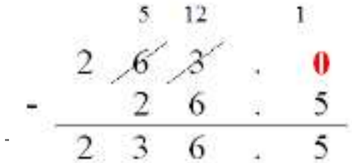
Children should practise subtracting numbers with both the same and different number of decimal places.

26.34 - 13.5 = 12.84

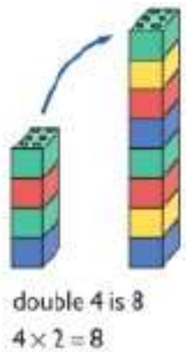
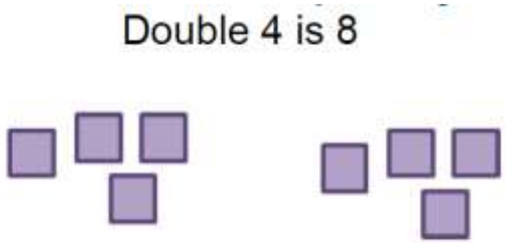
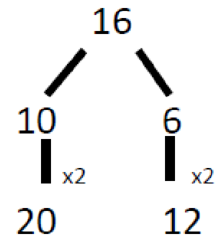
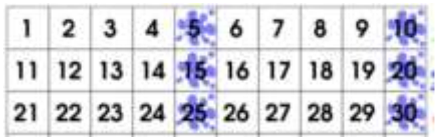


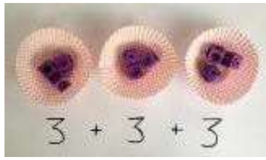
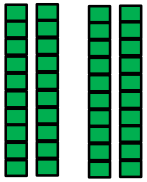
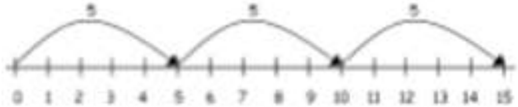
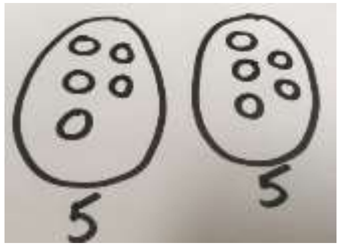
Children should be using rounding to estimate and the inverse to check.





Multiplication

	Strategy & Key Skills	Understanding	Application: Concrete	Application: Pictorial	Application: Abstract
YEAR 1 (EYFS)	*Doubling* • Count from 0 – 20. • Place numbers in order.	• Understand that a pair is a group of 2. • Understands that doubling is adding the same again. Key Vocab: Double, group, pair, half.	Use practical activities to show how to double a number.  Also introduce the vocabulary 'half' at this point. "The small tower is half the height of the big tower."	Draw pictures to show how to double a number. 	Mentally partition a number and then double each part before recombining it back together. 
YEAR 1	*Counting in multiples* • Count, read and write numbers to 100 in numerals (to 20 in words). • Count in multiples of 2s, 5s and 10s.	• Beginning to understand multiplication as equal groups of the same thing. • Understand what each digit represents in a teens number. • Supported using base 10, counters/cubes and bead strings begin to understand place value in numbers to and beyond 20. Key Vocab: Double, group, pair, multiple, times,	Count in multiples supported by concrete objects in equal groups.  	Use a number line, 100 square or pictures to continue support in counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10, 12, 14 5, 10, 15, 20, 25 , 30 10, ☐, 30, 40, 50, ☐

YEAR 2	*Repeated addition* • Read and write numbers to at least 100 in numerals and words. • Count forward and backward in 2s and 5s from 0. • Count forward and backward in 10s from any number. • Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.	• Understand place value in 2 digit numbers. • Understand multiplications as 'lots' or 'groups' of the same thing. • Understand and use the symbols × and = • <u>Key Vocab:</u> Lots/groups of, sets, times, multiply, divide, multiplied by, multiple.	Use different objects to add equal groups.   10 +10 + 10 + 10 (This can also be done using the place value counters)	Drawing pictures to represent problems: There are 3 plates. Each plate has 2 star biscuits on it. How many biscuits are there?  2 + 2 + 2 = 6 biscuits. 3 groups of 2 3 x 2 = 6 Using number lines: 5 + 5 + 5 = 3 × 5 = 3 groups of 5 = 15 	Write addition sentences and then multiplication to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10 5 x 2 = 10
YEAR 2	*Arrays- showing commutative multiplication* • Read and write numbers to at least 100 in numerals and words. • Count forward and backward in 2s, 3s and 5s from 0. • Count forward and backward in 10s from any number. • **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables**, including recognising odd and even numbers. • Know doubles to 20.	• Understand that multiplication is commutative, e.g. 4 × 3 = 3 × 4 • Make connections between the 10 multiplication table and place value and the 5 multiplication table • Understand and use the symbols × and = • Understand that division is the inverse of multiplication, e.g. 5 × 2 = 10, 2 × 5 = 10, 10 ÷ 2 = 5, 10 ÷ 5 = 2. <u>Key Vocab:</u> Multiply, inverse, partition, multiple, commutative.	Create arrays using counters/ cubes show multiplication sentences.  .	Draw the arrays to find the answer to the multiplication calculation. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 =15 3 x 5 = 15

YEAR 3

Grid Method to expanded column multiplication (2 digit times 1 digit)

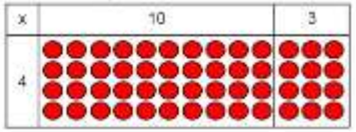
- Count, read and write numbers to 1000.
- Count from 0 in multiples of 4, 8, 50 and 100.
- Know doubles to 50.
- **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.**

- Understand place value in three-digit numbers.
- Understand how the inverse can be used to check answers.
- Understand the relationship between the 2, 4 and 8 multiplication tables.
- Understand how to multiply larger numbers by using related facts, e.g. 3 x 2 = 6 so 3 x 20 = 60.
- Use multiplication to solve scaling (eight times as long, four times as high) and correspondence problems (3 hats and 4 coats, how many different outfits?)

Key Vocab:
Multiply, times, multiplied by, inverse, partition, multiple, tens, ones

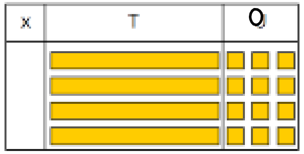
Show the link with arrays to first introduce the grid method.

13 x 4



4 rows of 10
4 rows of 3

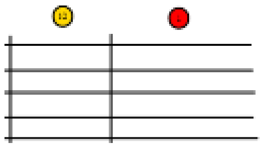
Move on to using Base 10 to move towards a more compact method.



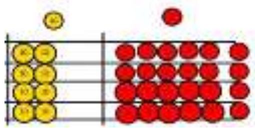
4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

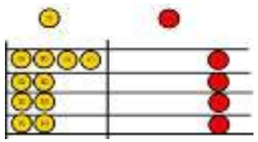
26 x 4



Fill each row with 26.



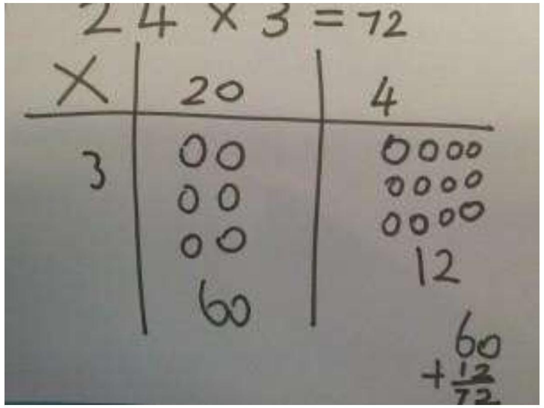
Add up each column, starting with the ones making any exchanges needed. **Totals should then be recorded under each column.**



Then you have your answer (104)

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Multiply by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

From here show children how to set this out using long multiplication, writing each calculation in brackets.

	3	5						
x		7						
	3	5	(	5	x	7	)	
2	1	0	(	3	0	x	7	)
2	4	5						

YEAR 4

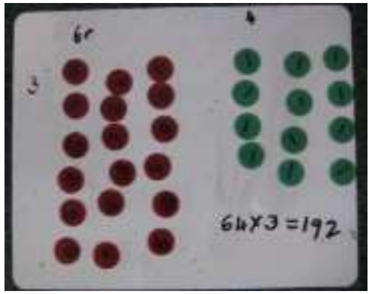
Grid Method – to short column multiplication (3 digit times 1 digit)

- Count, read and write numbers beyond 1000.
- Count in multiples of 6, 7, 9, 25 and 1000.
- Can double any 2 digit numbers.
- **Recall multiplication and division facts up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1 and multiplying three numbers.
  e.g. 2 x 6 x 5 =
  10 x 6 = 60

- Understand place value in four-digit numbers.
- Understand how the inverse can be used to check answers.
- Use rounding to estimate and check answers.
- Understand the distributive law, e.g. 39 x 7 = 30 x 7 + 9 x 7.
- Understand the associative law, e.g. (2 x 3) x 4 = 2 x (3 x 4)
- Understand how to multiply larger numbers by using related facts, e.g. 3 x 2 = 6 so 3 x 200 = 600.
- Use multiplication to solve scaling (eight times as long, four times as high) and correspondence problems (3 hats and 4 coats, how many different outfits?)

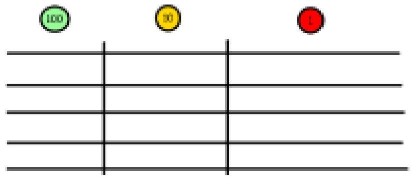
Key Vocab:
Multiply, carry, multiplied by, inverse, partition, multiple, thousands, hundreds, tens, ones, product.

Children can continue to be supported by place value counters at this stage of multiplication.

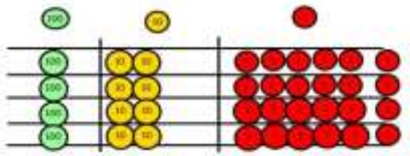


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

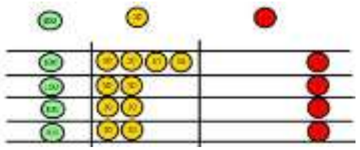
126 x 4



Fill each row with 126.



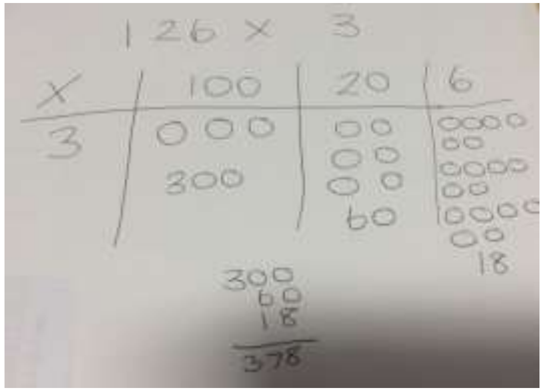
Add up each column, starting with the ones making any exchanges needed. **Totals should then be recorded under each column.**



Then you have your answer (504)

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Showing the clear addition alongside the grid.

535 x 7 Estimate: 550 x 7 = 3850

x	500	30	5
7	3500	210	35

3500 + 210 + 35 = 3745

From here show children how to set this out using long multiplication, writing each calculation in brackets.

```
    h t o
    5 3 5
x       7
      3 5 ( 5 x 7 )
    2 1 0 ( 3 0 x 7 )
  3 5 0 0 ( 5 0 0 x 7 )
  3 7 4 5
```

When confident, then children can move onto short multiplication.

```
    h t o
    5 3 5
x       7
  3 7 4 5
    2 3
```

"Next I need to multiply the 3 **tens** by 7"

YEAR 5 AND BEYOND

Column method- 4 digit x 2 digit and 2 decimal places by 1 digit.

- Count, read and write numbers beyond 1,000,000 (up to 10,000,000 for Year 6).
- Identify multiples and factors of two numbers.
- Recall prime numbers to 19.
- Multiply and divide numbers mentally drawing upon know facts.
- Multiply and divide whole numbers by 10, 100 and 1000.
- Recognise and use square and cube numbers and the notation for squared (^{2}) and cubed (^{3}).

- Understand place value in numbers beyond four-digits (up to 10,000,000 for Year 6).
- Understand how the inverse can be used to check answers.
- Use rounding to estimate and check answers.

Key Vocab:

Prime, prime factor, common factor, common multiple, composite numbers, multiple, factor, square number, squared (^{2}), cubed number, cubed (^{3}), carry, tenths, hundredths, decimal.

Understand the use of brackets and the associative law of addition.

e.g. (a + b) + c = a + (b + c)

(1 + 2) + 5 = 1 + (2 + 5)

Begin to understand the order of operations – **BODMAS.**

B = Brackets

O = Orders (powers and square roots)

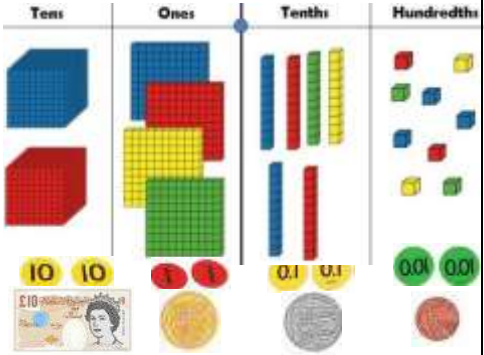
DM = Division and Multiplication

AS = Addition and Subtraction

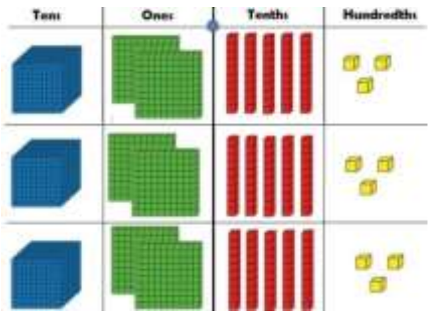
e.g. 50 + 15 ÷ 5 (÷ before +)

50 - 3 = 53

Concrete materials should be used when introducing the multiplication of decimals. Start with base 10 (reassigning the values) on laminated place value grids (T O . t h) and then move to place value counters and using money.

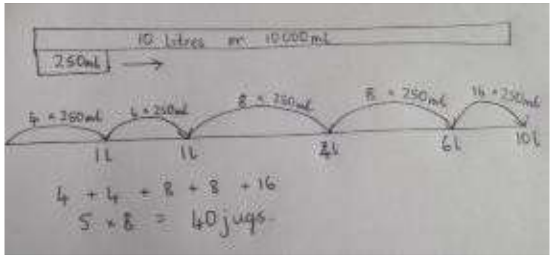


12.53 x 4



And then making exchanges as per the previous page.

Number lines can support learners when solving problems with multiplication alongside the formal written methods.



Multiply by two digit numbers and showing the clear addition alongside the grid.

35 x 77 Estimate: 30 x 80 = 2400

×	30	5	
7	210	35	2450
7	210	35	245

2450 + 245 = 2695

From here show children how to set this out using expanded long multiplication, writing each calculation in brackets.

		3	5							
×		7	7							
		3	5	(	5	×	7	)		
	2	1	0	(	3	0	×	7	)	
	3	5	0	(	5	×	7	0	)	
2	1	0	0	(	3	0	×	7	0	)
2	6	9	5							

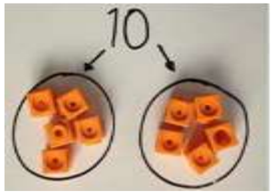
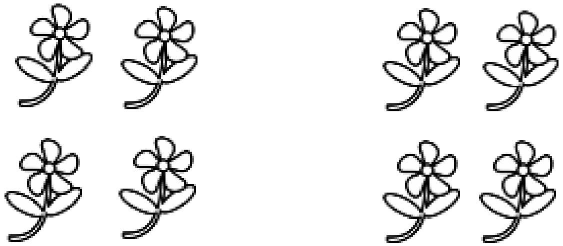
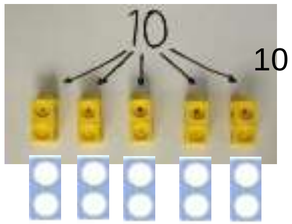
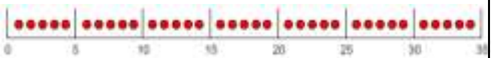
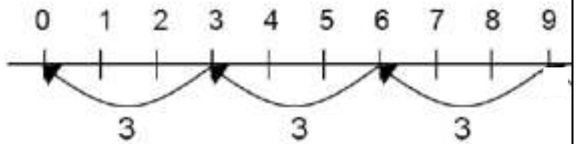
This then moves to the more compact method.

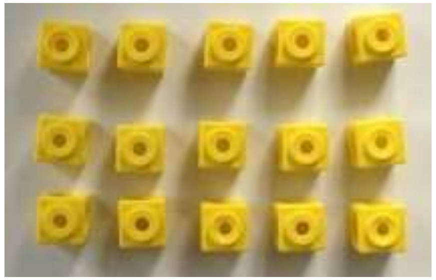
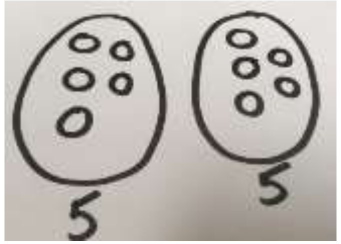
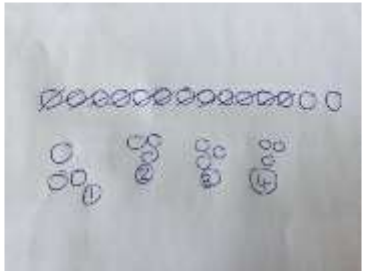
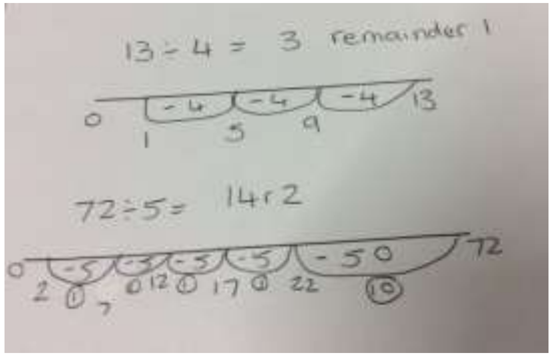
		3	5							
×		7	7							
	2	4 [3]	5	(	3	5	×	7	)	
2	4 [3]	5	0	(	3	5	×	7	0	)
2	6	9	5							

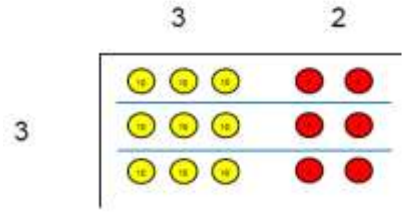
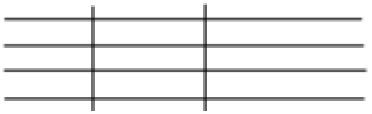
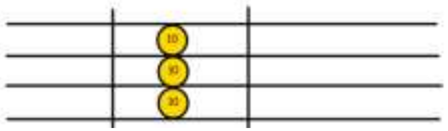
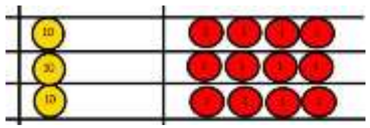
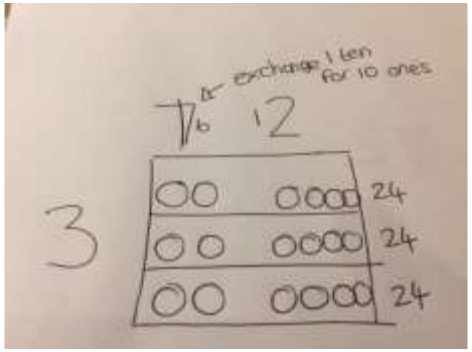
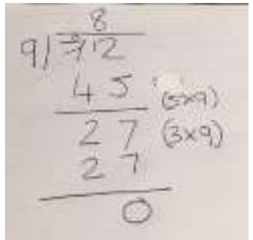
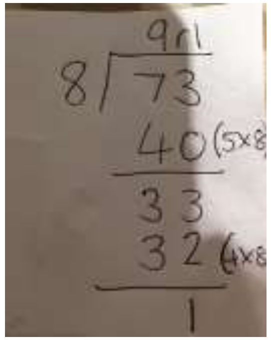
Decimals:

	1	8	.	7
×				8
1	4	9	.	6
	6	5		

Division

	Strategy & Key Skills	Understanding	Application		
			Concrete	**Pictorial**	**Abstract**
YEAR 1 (EYFS)	*Sharing* • Count from 0 – 20. • Place numbers in order.	• Understand that halving is sharing into two equal groups. Key Vocab: Double, half, share, equal.	I have 10 cubes. Can you share them equally between 2 groups? 10 ÷ 2 = 5   	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 10 cakes equally between five people. 10 ÷ 5 = 2 cakes
YEAR 1	*Division as grouping* • Count, read and write numbers to 100 in numerals (to 20 in words). • Count in multiples of 2s, 5s and 10s.	• Understand what each digit represents in a teens number. • Supported using base 10, counters/cubes and bead strings begin to understand place value in numbers to and beyond 20. • Understand how to find a half and a quarter of an object, shape or quantity. Key Vocab: Equal groups, double. half.	Divide quantities into equal groups. Use cubes, counters, objects to aid understanding. How many groups of two are in ten?  10 ÷ 2 = 5 "There are five groups of two in ten." 5 x 2 = 10 Divide 35 into groups of 5. 	As above children use pictures or shapes to share quantities. Progress to using a number line to show jumps in groups. The number of jumps equals the number of groups. Remind that we are counting backwards down the numberline as we are subtracting. How many groups of three are in nine? 9 ÷ 3 = 3  At this point talk to the children about what they are doing: they are taking away groups of 3 to see how many are in 9.	If there are 10 cakes, how many people can have 2 cakes each? 10 ÷ 2 = 5 people How many groups of 2 can you make out of 8? $\square$ x 2 = 8 8 ÷ 2 = 4

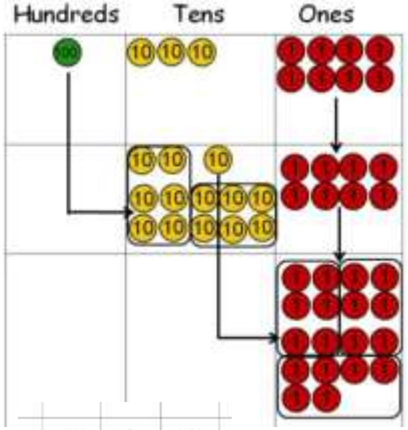
YEAR 2	*Division within arrays* • Read and write numbers to at least 100 in numerals and words. • Count forward and backward in 2s and 5s from 0. • Count forward and backward in 10s from any number. • **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables**, including recognising odd and even numbers.	• Understand place value in 2 digit numbers. • Understand that division is NOT commutative, e.g. 20 ÷ 4 ≠ 20 ÷ 4 • Understand and use the symbols ÷ and = • Understand that multiplication is the inverse of, division e.g. 4 × 3 = 12, 3 × 4 = 12, 12 ÷ 3 = 4, 12 ÷ 4 = 3. <u>Key Vocab:</u> Divide, divided by, share equally, equal groups, halve, half, inverse.	Link division to multiplication by creating an array and thinking about the number sentences that can be created  E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15 At this point discuss whether division is commutative. Could we have 3 ÷ 15? Is it possible to share 3 between 15 people?	Children to draw the correct number of groups and share the amount equally between each group.  E.g. 10 ÷ 2 = 5 2 x 5 = 10 10 ÷ 5 = 2 5 x 2 = 10	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 How would we solve this? ☐ ÷ 2 = 12 2 x 12 = 24
YEAR 3	*Division with a remainder* • Count, read and write numbers to 1000. • Count from 0 in multiples of 4, 8, 50 and 100. • Know doubles to 50. • **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.**	• Understand place value in three-digit numbers. • Understand how the inverse can be used to check answers. • Understand the relationship between the 2, 4 and 8 multiplication tables. • Understand how to multiply larger numbers by using related facts, e.g. 6 ÷ 2 = 3 so 60 ÷ 20 = 3 and 60 ÷ 2 = 30. <u>Key Vocab:</u> divisor, quotient, remainder.	Divide objects between groups and see how much is left over. 14 ÷ 3 = 4 remainder 2 	Use repeated subtraction on a number line. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder Using the inverse to check: 8 x 3 = 24 + 5 = 29

YEAR 3	*Arrays to short division* *(2 digit divided by 1 digit)* • Count, read and write numbers to 1000. • Count from 0 in multiples of 4, 8, 50 and 100. • Know halves to 100. • **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.**	• Understand place value in three-digit numbers. • Understand how the inverse can be used to check answers. • Understand the relationship between the 2, 4 and 8 multiplication tables. • Understand how to divide larger numbers by using related facts, e.g. 6 ÷ 2 = 3 so 60 ÷ 20 = 3 and 60 ÷ 2 = 30. <u>Key Vocab:</u> divisor, quotient, remainder, exchange, hundreds, tens, ones.	Show the link with arrays to first introduce the method of short division using base 10 and then place value counters (bus stop). 96 ÷ 3 = 32  42 ÷ 3 = Use place value counters to divide using the bus stop method alongside   Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much is in 1 group so the answer is 14.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 72 ÷ 3 = 24 	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Continue to encourage mental methods. Some children may be able to do calculations such as these in their head by this point.

<table>
<tr>
<td>YEAR 4</td>
<td>Long division
(3 digit divided by 1 digit)

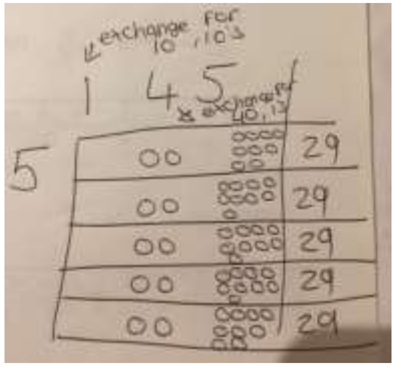
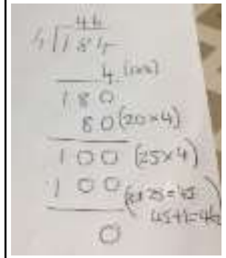
• Count, read and write numbers beyond 1000.
• Count in multiples of 6, 7, 9, 25 and 1000.
• Can halve any 2 digit numbers.
• Recall multiplication and division facts up to 12 x 12.
• Recognise and use factor pairs in mental calculations.
• Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1 and multiplying three numbers.</td>
<td>• Understand place value in four-digit numbers.
• Understand how the inverse can be used to check answers.
• Use rounding to estimate and check answers.
• Understand how to divide larger numbers by using related facts,
e.g. 6 ÷ 2 = 3 so
600 ÷ 200 = 3
600 ÷ 20 = 30
600 ÷ 2 = 300.

<u>Key Vocab:</u>
remainder, exchange, thousands, hundreds, tens, ones.</td>
<td>Children can continue to be supported by place value counters at this stage of division. This can be recorded in children's books as shown below.

138 ÷ 6 = 23


<table>
<tr><td></td><td>0</td><td>2</td><td>3</td></tr>
<tr><td>6</td><td>1</td><td>3</td><td>8</td></tr>
<tr><td></td><td>1</td><td>2</td><td>0</td></tr>
<tr><td></td><td></td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>0</td></tr>
</table>
Start with the hundreds. Can this be split into groups of 6? No, so exchange for 10 tens. Next look at the tens. How many groups of 6 can you make? Two groups can be made so write a 2 in the tens columns above the line. Remove these two groups of 6 tens (2 x 60 = 120) and write this below 138. You now have 1 ten and 8 ones left. The tens cannot be split into groups of 6 so this needs to be exchanged for 10 ones. The 18 ones can be split into 3 groups of 6 so this is written in the ones column above the line. When these groups are removed there are no counters left, leaving a remainder of 0.</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown. Progressing to the more formal method opposite.

</td>
<td>Start by using the method in Y3.


138 ÷ 6 = 23
<table>
<tr><td></td><td>0</td><td>2</td><td>3</td></tr>
<tr><td>6</td><td>1</td><td>3</td><td>8</td></tr>
<tr><td></td><td>1</td><td>2</td><td>0</td></tr>
<tr><td></td><td></td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>0</td></tr>
</table>
Move onto divisions with a remainder.
<table>
<tr><td></td><td>1</td><td>5</td><td>5</td></tr>
<tr><td>3</td><td>4</td><td>6</td><td>7</td></tr>
<tr><td></td><td>3</td><td>0</td><td>0</td></tr>
<tr><td></td><td>1</td><td>6</td><td>7</td></tr>
<tr><td></td><td>1</td><td>5</td><td>0</td></tr>
<tr><td></td><td></td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>1</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td>2</td></tr>
</table></td>
</tr>
</table>

YEAR 5 AND BEYOND

Column method- 4 digit x 2 digit and 2 decimal places by 1 digit.

- Count, read and write numbers beyond 1,000,000 (up to 10,000,000 for Year 6).
- Identify multiples and factors of two numbers.
- Recall prime numbers to 19.
- Multiply and divide numbers mentally drawing upon know facts.
- Multiply and divide whole numbers by 10, 100 and 1000.
- Recognise and use square and cube numbers and the notation for squared (^{2}) and cubed (^{3}).

- Understand place value in numbers beyond four-digits (up to 10,000,000 for Year 6).
- Understand how the inverse can be used to check answers.
- Use rounding to estimate and check answers.

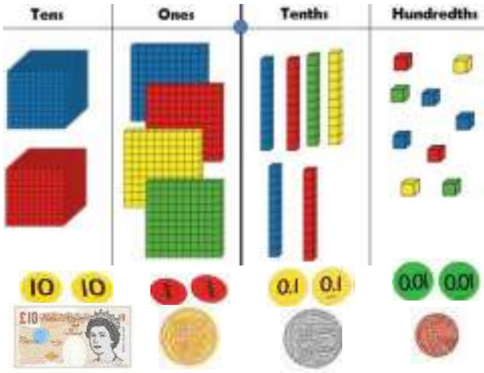
Key Vocab:
Prime, prime factor, common factor, common multiple, composite numbers, multiple, factor, square number, squared (^{2}), cubed number, cubed (^{3}), carry, tenths, hundredths, decimal.

Understand the use of brackets and the associative law of addition.
e.g. (a + b) + c = a + (b + c)
(1 + 2) + 5 = 1 + (2 + 5)

Begin to understand the order of operations – **BODMAS.**
B = Brackets
O = Orders (powers and square roots)
DM = Division and Multiplication
AS = Addition and Subtraction

e.g. 50 + 15 ÷ 5 (÷ before +)
50 - 3 = 53

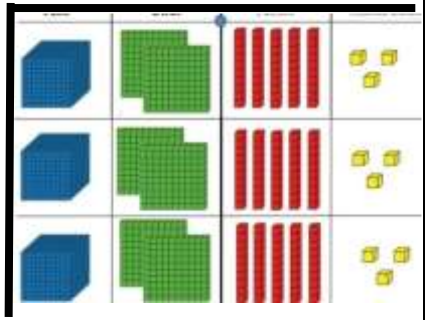
Concrete materials should be used when introducing the division of decimals. Start with base 10 (reassigning the values) on laminated place value grids (T O . t h) and then move to place value counters and using money.



37.59 ÷ 3 = 12.53

1 2 . 5 3

3



Starting with sharing the tens, then the ones etc. in the same way as whole numbers. See the previous example.

Number lines can support learners when solving problems with multiplication alongside the formal written methods.

Estimate: 6000 ÷ 8 = 750

	0	7	6	2
8	~~6~~	^{6}0	^{4}9	^{1}6

		2	8	r 12
1 5	4	3	2	
	3	0	0	
	1	3	2	
	1	2	0	
		1	2	

Interpret remainders as whole number remainders, fractions, or by rounding, depending on the context.

3018 ÷ 8

	0	3	7	7	.	2	5
8	~~3~~	^{3}0	^{6}1	^{5}8	.	^{2}0	^{4}0

or 377 r 2 **or** 377 $^2/_8$

Decimals:

	0	2	4	7	.	7	9
5	~~1~~	^{1}2	^{2}3	^{3}8	.	^{3}9	^{4}5